Quantum and reversible logic circuits have found emerging attentions in nanotechnology, optical computing, quantum computing and low power CMOS design. In this paper we propose some new quantum and reversible compressors using our new genetic algorithm based simulator, analyzer and synthesizer software. In the best of our knowledge only reversible 4:2 compressor is designed before in the literature. The proposed quantum 4:2 compressor circuits are compared with the existing counterparts in terms of number of constant inputs, number of garbage outputs, delay and the quantum cost. We have also designed quantum 6:2 and 7:2 compressors for the first time. They are optimized in terms of quantum cost and delay. The proposed designs can be used as a basic block in complex systems like multipliers, and can execute complicated operations better than the existing designs in literatures.

Keywords: Quantum computing, nanotechnology based systems, simulator, quantum circuits, quantum gates, software, synthesis, optimization, analysis, reversible logic, compressor, adders

1. Introduction

Energy loss is an important consideration in nanotechnology and low power CMOS circuit design. In 1961 it is proved by Landauer that irreversible logic results in energy dissipation regardless of its underlying technology due to the information loss [1]. The amount of energy dissipated for every irreversible bit operation is equal to $kT \ln 2$ Joules, where $k=1.3806505 \times 10^{-23}$ m$^2$kg$^{-1}$K$^{-1}$ (Joules Kelvin$^{-1}$) is the Boltzmann’s constant and $T$ is the absolute temperature at which operation is performed [2].

Reversible logic circuits are those circuits that do not lose any information because they allow the reproduction of inputs from observed outputs. In every reversible logic circuit there is a one-to-one mapping between input and output vectors. It is proved by Bennett that $kT \ln 2$ Joules of energy would not dissipate from a circuit if the circuit is reversible [2].

Quantum and Reversible circuits on the other hand have found promising attention in nanotechnology 0[3], quantum computing [4], and low power CMOS circuit design. In the past decade a few reversible logic circuit synthesis methods are proposed [5,6]. We can use Genetic Algorithm (GA) to synthesize and optimize reversible logic circuits as well as quantum logic circuits [7,8,9]. GA-based methods have the ability to manage don’t care conditions (DCs) [9,10].

In this paper some quantum and reversible compressors are designed and optimized using our new developed software. The proposed quantum and reversible 4:2 compressors are compared with the existing counterparts and the results are reported. Each proposed design is better than the existing designs in terms of at least one important parameter in designing quantum and reversible circuits like number of constant inputs, number of garbage outputs, quantum cost, delay and size of the circuit. On the other hand, we proposed quantum 6:2 and 7:2 compressors for the first time.

The paper is organized as follows: First we explain the background including compressor, and quantum and reversible logic circuits and gates. Then the design and implementation of quantum circuit simulator, analyzer and synthesizer...
(GQRNS) software is explained in section 3. We named the software “GQRNS” which stands for “Genetic Quantum Reversible Nano Synthesis Simulation Software”. Genetic algorithm is used in the software. Thus the genetic algorithm and its application in the synthesis of quantum and reversible circuits are also provided in section 3. Then the proposed quantum and reversible compressors using our new simulator, analyzer and synthesizer software is presented in section 4. Evaluations of the proposed circuits are provided in section 5. Conclusions and a comprehensive list of references are also provided.

2. Background
In this section brief background information about compressor and quantum and reversible gates and circuits are presented.

2.1. Compressors:
An \((n:m)\) compressor is a variant of a counter with \(n\) primary inputs, with the same weights \(2^i\), and \(m\) primary outputs of weight \(2^i, 2^{i+1}, \ldots, 2^{i+m-1}\). It is to be noted that the compressor has some incoming carries of the weight \(2^i\) from previous compressors and some outgoing carries with the weights \(2^{i+1}\) and up [26].

The 4:2 compressor block was introduced in 1981 [11]. Its functionality is that it compresses five bits into three. Four of the input bits have the same bit position \((2^n)\) and the other bit is fed from the previous position \((2^{n-1}\) or carry-in). The 4:2 compressor has actually three outputs: one output is in the position \(n\) and two outputs are in the position \(n+1\). A traditional 4:2 compressor block diagram is depicted in Figure 1. The 4:2 compressor consists of two full adders.

![Fig. 1: Traditional 4:2 Compressor Building Block [25]](image)

2.2. Quantum and Reversible Gates and Circuits:
A gate (circuit) is reversible if and only if there is a one-to-one mapping between its inputs and outputs [5,12]. Reversible logic gates can be implemented in various technologies such as CMOS, optical, quantum and nanotechnology. Quantum gates (circuits) act on qubits. A qubit is a unit of quantum information [9,12]. Since quantum gates cannot be implemented using conventional technologies such as CMOS, some other new technologies such as NMR, Ion-Trap and QCA are developed in the past decades.

A quantum (reversible) gate has the same number of inputs and outputs. Neither feedback nor fanout are permitted in quantum (reversible) circuits. There exists many quantum gates such as Feynmann [12], Toffoli [5], Fredkin [13], Peres [14], Hadamard [12], V [12], V+ [12], NFT [15], TSG [16], HNFG [17], MKG [18], HNG [17].

Some two qubit quantum gates are shown in Fig. 2. The 2×2 Feynman gate also known as controlled NOT (CNOT) is also depicted in Fig.2.a. If the control input of CNOT is set to ‘0’, the gate acts as a BUFFER gate; else, it acts as a NOT gate. The Feynman gate can be used as a copying circuit to provide a copy of the signal. If the \(B\) input in Fig.1.a is set to ‘0’ then two outputs of the Feynman gate are equal to ‘\(A\)’ input. The \(V\) gate also named as square root of NOT gate (\(√\text{NOT}\), is shown in Fig.2.b. \(V^*\) gate is the complex conjugate transpose of \(V\) (See Fig.2.c.). The \(V\) and \(V^*\) quantum gates have some properties that are shown in Eq. 1.

\[
\begin{align*}
V \times V &= \text{NOT} \\
V \times V^* &= V^* \times V = I \\
V^* \times V^* &= \text{NOT}
\end{align*}
\]

These equations show that two \(V\) gates in series or two \(V^*\) gates in series are equivalent to the NOT gate; and two \(V\) and \(V^*\) in series, are equivalent to an identity or a BUFFER gate.

Toffoli and Fredkin gates [5,13] which are depicted in Fig.3.a,b are 3×3 reversible gates. Both of them are universal, i.e. any logical reversible circuit can be implemented using these gates.
Fig. 2 some quantum two qubit gates (a) Feynman gate (b) Controlled $V$ gate (c) Controlled $V^+$ gate.

Fig. 3 Most common three qubit quantum gates (a) Toffoli gate, (b) Fredkin gate

Many other useful quantum three qubit gates are also proposed in the literature. Figures 4.a and 4.b show the New Gate (NG) and the Peres gates, respectively.

Fig. 4 (a) New gate (b) Peres gate

The TSG, MKG and HNG gates are well known $4 \times 4$ reversible gates. These gates and their functionalities are shown in Fig. 5.

Fig. 5 Some $4 \times 4$ reversible gates: (a) TSG, (b) MKG and (c) HNG.

The quantum cost (QC) of a reversible circuit is defined as the number of $1 \times 1$ or $2 \times 2$ reversible quantum or logic gates that are needed to realize the circuit [12]. For instance, the QC of Toffoli, Fredkin, Peres, TSG, MKG, and HNG are 5, 5, 4, 13, 13, and 6 respectively.
3. Design and Implementation of Quantum Circuit Simulator, Analyzer and Synthesizer (GQRNS\(^3\) software) in Nanotechnology

To develop the GQRNS\(^3\) software we use Genetic Algorithm (GA). In GA synthesis, variables of search space have to be coded to a string of bits, named chromosome. Each gate is coded as shown in Fig.6. Since various gate types are used in the synthesis, we consider a field as the gate code and assign one binary number to each type of gates.

<table>
<thead>
<tr>
<th>Gate code</th>
<th>Target Output</th>
<th>Control 1</th>
<th>Control 2</th>
<th>…</th>
<th>Control n</th>
</tr>
</thead>
</table>

Fig.6. Coding of quantum and reversible controlled gates in the GQRNS\(^3\)

The second field represents the location of target output of the gate. The remaining fields are location of control inputs of the gate. In our software, each chromosome represents a complete circuit and includes the codes of \(m\) gates and the constant inputs of the circuit (\(C_1\) to \(C_n\)) depicted in Fig.7.

<table>
<thead>
<tr>
<th>(C_1)</th>
<th>…</th>
<th>(C_n)</th>
<th>Gate 1</th>
<th>Gate 2</th>
<th>…</th>
<th>Gate (m)</th>
</tr>
</thead>
</table>

Fig.7. Structure of a chromosome in GQRNS\(^3\) software which codes a complete quantum and reversible circuit.

The flow chart of the GQRNS\(^3\) synthesis part is shown in Fig.8. First, we define a population of chromosomes and apply three basic operators of GA to each chromosome of the population. The \textit{mutation} operator selects one chromosome randomly and applies random changes to the selected chromosome with a specific probability. The \textit{crossover} operator selects two chromosomes randomly and exchanges their corresponding segments. The \textit{selection} operator selects some of proper chromosomes for reproduction of the next population. In the GQRNS\(^3\), each generated chromosome is a coded circuit.

![Fig.8: The GQRNS\(^3\) software synthesis algorithm.](image-url)
The proposed GQRNS³ software has the ability to use the Toffoli, Peres, or Fredkin gates or a combination of them for the synthesis of quantum and reversible circuits. Another gate library in GQRNS³ is a set of universal quantum gates including V, V⁺ and Feynman gates. The GQRNS³ is able to synthesize a given function, in truth table form, using each of above mentioned gates separately, or simultaneously. In GQRNS³, one can define a desired quantum or reversible function with or without don’t care (DC) inputs, DC outputs or DC conditions. In [10], the “don’t care” values in a reversible function or circuit are classified into three types: DC inputs, DC conditions and DC outputs. In our software we also use the same definition of don’t cares as described in [10].

4. New Designs of quantum compressors

In this section some quantum and reversible compressors are proposed. Some of them are synthesized and optimized using the GQRNS³ software.

4.1. Our Proposed reversible 4:2 Compressors:

As mentioned before, the 4:2 compressor has four main inputs and two main outputs with one carry input and one carry output. It can be implemented using two full adder blocks. In reversible circuits, the MKG gate, presented in [18], can be used as the reversible full adder block. Therefore, in design #1 we use two MKG gates. The proposed design is depicted in Fig. 9. Its QC is 26 and it requires two constant inputs. Number of garbage outputs in this design is 4. The delay of the design #2 is 20.

![Fig.9. Our proposed (Design #1) reversible 4:2 compressor.](image)

In design #2 we use HNG gate which has optimum QC between all reversible full adders [20]. The proposed design is depicted in Fig. 10. Its QC is 12 and it requires two constant inputs and generates four garbage outputs. The delay of the design #2 is 12.

![Fig.10. Our proposed (Design #2) reversible 4:2 compressor.](image)
Using the GQRNS\(^3\) we obtain a quantum circuit with 4 Peres gates which has a QC of 16. The proposed design is depicted in Fig. 11. It has two constant inputs and four garbage outputs. Since the delay of the Peres gate is 3 [27], the overall circuit has a delay of 12.

![Fig. 11. Our proposed (Design #3) quantum 4:2 compressor using GA-based Synthesizer in nanotechnology.](image)

Using a different library of gates in the GQRNS\(^3\) we obtain a better design. This design has a QC of 12. The proposed design is depicted in Fig. 12. It has two constant inputs and four garbage outputs. Although the QC of design #2 and #4 are the same, the delay of the design #4 is less than that of the design #2. The HNG gate has delay of 6; therefore the delay of the reversible 4:2 compressor of design #2 is 12. The delay of design #4 which is generated by GQRNS\(^3\), is only 7. Thus we can see that the design #4 is the best design among our proposed quantum and reversible 4:2 compressors.

![Fig. 12. Our proposed (Design #4) quantum 4:2 compressor using GA-based Synthesizer in nanotechnology.](image)

4.2. Our proposed quantum 6:2 compressors
The 6:2 compressor has six main inputs (order of 2\(^6\)) and two main outputs with three carry inputs and three carry outputs. It can be synthesized using GQRNS\(^3\) software. We have two different designs for this circuit. In design #1 our gate library uses only Peres gates. The proposed design is depicted in Fig. 13. Its QC is 32 and it requires four constant inputs. Number of garbage outputs in this design is 8. The delay of this design is 15.

![Fig. 13. Our proposed (Design #1) quantum 6:2 compressor using GA-based Synthesizer in nanotechnology. Its delay is 18 and its QC is 32.](image)
In the second approach we use the different gate library including quantum 2-qubit V, V’, and CNOT gates. The design #2 is shown in Fig.14. It has QC of 24 and delay of only 10. Number of constant inputs and garbage outputs are 4 and 8, respectively. Thus, it is better than design #1 (Fig.13) in terms of QC and delay.

Fig.14. Our proposed (Design #2) quantum 6:2 compressor using GA-based Synthesizer in nanotechnology. Its delay is only 10 and its QC is 24.

4.3 Our proposed quantum 7:2 compressors:
The 7:2 compressor has seven main inputs (order of 2^n) and two main outputs with four carry inputs and four carry outputs. It can be synthesized using GQRNS software. The gate library which is used for this compressor design includes quantum 2-qubit V, V’, and CNOT gates. This design is depicted in Fig.15. It has QC of 30 and delay of only 9. Number of constant inputs and garbage outputs are 5 and 10, respectively.

Fig.15. Our proposed (Design #6) reversible 7:2 compressor using GA-based Synthesizer in nanotechnology. Its delay is only 9 and QC is 30.
5. Evaluation of the proposed quantum and reversible compressor circuits

In the best of our knowledge, the quantum 6:2 and 7:2 compressors are not proposed in the literature. Thus, we only present the specifications of these quantum compressors, as shown in the table 1.

The proposed quantum 4:2 compressor circuits are more efficient than the existing circuits presented in [16,21,22,23,24]. Evaluation of the proposed circuits can be comprehended easily with the help of the comparative results in table 2.

Table1: Specifications of our proposed quantum 6:2 and 7:2 compressors

<table>
<thead>
<tr>
<th></th>
<th>Gin</th>
<th>Gout</th>
<th>QC</th>
<th>Delay</th>
</tr>
</thead>
<tbody>
<tr>
<td>6:2 Compressor (Design #1)</td>
<td>4</td>
<td>8</td>
<td>32</td>
<td>15</td>
</tr>
<tr>
<td>6:2 Compressor (Design #2)</td>
<td>4</td>
<td>8</td>
<td>24</td>
<td>10</td>
</tr>
<tr>
<td>7:2 Compressor</td>
<td>5</td>
<td>10</td>
<td>30</td>
<td>9</td>
</tr>
</tbody>
</table>

Table2: Comparative experimental results of different quantum and reversible 4:2 compressors

<table>
<thead>
<tr>
<th></th>
<th>Gin</th>
<th>Gout</th>
<th>QC</th>
<th>Delay</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1</td>
<td>2</td>
<td>4</td>
<td>26</td>
<td>20</td>
</tr>
<tr>
<td>#2</td>
<td>2</td>
<td>4</td>
<td>12</td>
<td>12</td>
</tr>
<tr>
<td>#3</td>
<td>2</td>
<td>4</td>
<td>16</td>
<td>12</td>
</tr>
<tr>
<td>#4</td>
<td>2</td>
<td>4</td>
<td>12</td>
<td>7</td>
</tr>
<tr>
<td>[22,23]</td>
<td>2</td>
<td>4</td>
<td>13</td>
<td>NA</td>
</tr>
<tr>
<td>[16]</td>
<td>2</td>
<td>4</td>
<td>26</td>
<td>NA</td>
</tr>
<tr>
<td>[21]</td>
<td>4</td>
<td>6</td>
<td>30</td>
<td>NA</td>
</tr>
<tr>
<td>[24]</td>
<td>8</td>
<td>10</td>
<td>25</td>
<td>NA</td>
</tr>
</tbody>
</table>

As we can see in table 2, the design #4 is the best design because it has minimum number of constant inputs, garbage outputs, QC, and delay.

Conclusion

In this paper we propose some new quantum and reversible compressors using our new genetic algorithm based simulator, analyzer and synthesizer software. We have developed and used a genetic algorithm-based simulator, analyzer and synthesizer software. The mentioned software has an extensive library of quantum nano gates which results in finding optimized quantum and reversible logic designs. In the best of our knowledge only reversible 4:2 compressor is designed before in the literature. The proposed quantum 4:2 compressor circuits are better than the existing counterparts in terms of at least one of the main factors such as number of constant inputs, number of garbage outputs, delay and the quantum cost. We have also designed quantum 6:2 and 7:2 compressors using our new software, GQRNS3 for the first time. They are optimized in terms of quantum cost and delay. The proposed designs can be used as a basic block in complex systems like multipliers, and can execute complicated operations. This paper will be a good start towards design of other quantum and reversible logic compressors.

References