Heat Transfer

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Chapter 13

Radiation Exchange Between Surfaces

Today’s Topics

- Radiation Exchange Between Opaque, Diffuse, Gray Surfaces in an Enclosure
- Net Radiation Exchange at a Surface
- Radiation Exchange Between Black Surfaces
- Radiation Exchange Between Gray Surfaces
Radiation Exchange Between Opaque, Diffuse, Gray Surfaces in an Enclosure

- All surfaces of the enclosure are opaque, diffuse, gray
- The medium is assumed to be transparent
- Each surface of the enclosure is assumed to be isothermal with uniform irradiation
- The temperature or radiative heat flux associated to each surface is known
- The objective is to determine the unknown radiative heat fluxes or temperatures associated with each of the surfaces

Net Radiation Exchange at a Surface

\[ q_i = A_i (J_i - G_i) \]
\[ J_i = E_i + \rho_i G_i = \varepsilon_i E_b + (1 - \alpha_i) G_i = \varepsilon_i E_b + (1 - \varepsilon_i) G_i \]
Net Radiation Exchange at a Surface

\[ J_i = \varepsilon_i E_{bi} + (1 - \varepsilon_i)G_i \]

\[ G_i = \frac{J_i - \varepsilon_i E_{bi}}{(1 - \varepsilon_i)} \]

\[ q_i = A_i (J_i - G_i) = A_i \left( J_i - \frac{J_i - \varepsilon_i E_{bi}}{(1 - \varepsilon_i)} \right) \]

\[ q_i = \frac{E_{bi} - J_i}{(1 - \varepsilon_i) / (\varepsilon_i A_i)} \]

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\[ A_i G_i = F_{i1} A_i J_1 + F_{i2} A_i J_2 + \cdots + F_{iN} A_i J_N = \sum_{j=1}^{N} F_{ij} A_i J_j \]

From reciprocity

\[ A_i G_i = \sum_{j=1}^{N} F_{ij} A_i J_j = \sum_{j=1}^{N} F_{ij} J_j = A_i \sum_{j=1}^{N} F_{ij} J_j \]
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\[ q_i = A_i (J_i - G_i) = A_i (J_i - \sum_{j=1}^{N} F_{ij} J_j) \]

\[ q_i = A_i (J_i \sum_{j=1}^{N} F_{ij} - \sum_{j=1}^{N} F_{ij} J_j) = A_i \left( \sum_{j=1}^{N} F_{ij} J_i - \sum_{j=1}^{N} F_{ij} J_j \right) \]

From summation rule

\[ q_i = \sum_{j=1}^{N} A_i F_{ij} (J_i - J_j) = \sum_{j=1}^{N} q_{ij} \]

\[ q_i = \frac{E_{bi} - J_i}{(1 - \varepsilon_i) / (\varepsilon_i A_i)} = \sum_{j=1}^{N} \frac{J_i - J_j}{(A_i F_{ij})^2} \]

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\[ q_j = \frac{E_{bi} - J_j}{(1 - \varepsilon_j) / (\varepsilon_j A_j)} = \sum_{i=1}^{N} \frac{J_i - J_j}{(A_i F_{ij})^2} \]
Radiation Exchange Between Surfaces

(Example)

Known: A curved, solar absorber surface with a special coating is being cured by use of an infrared heater in a large room

Find: Net rate of heat transfer to the absorber surface

Assumptions: (1) Steady-state condition, (2) convection effects are negligible, (3) Absorber and heater surfaces are diffuse and gray, (4) Surrounding room is large

\[
\begin{align*}
T_3 &= T_\text{sur} = 300 \text{ K} \\
\varepsilon_3 &= 1 \\
H &= 1 \text{ m} \\
W &= 1 \text{ m} \\
T_2 &= 600 \text{ K}, A_2 = 25 \text{ m}^2, \\
\varepsilon_2 &= 0.5 \\
T_1 &= 1000 \text{ K}, A_1 = 10 \text{ m}^2, \\
\varepsilon_1 &= 0.9 \\
L &= 10 \text{ m} \\
\end{align*}
\]

Diagram:

1. Collector, \( L = 10 \text{ m} \)
2. Heaters, \( L = 10 \text{ m} \)
3. Absorber surface
4. Infrared heater
5. Surrounding room

Equations:

\[
\begin{align*}
F_{12} &= F_{12}' = 0.39 \\
F_{11} + F_{12} + F_{13} &= 1 \\
A_2 F_{23} &= A_3 F_{32} = A_4 F_{42} = A_2 F_{23} = A_4 F_{13} \\
A_2 F_{23} &= \frac{A_1 F_{13}}{A_2} = \frac{10}{15} \times 0.61 = 0.41 \\
E_{63} &= \sigma T_3^4 \\
E_{63} &= \sigma T_4^4 \\
E_{63} &= \sigma T_2^4 \\
\end{align*}
\]
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\[
q_1 = \frac{E_{b1} - J_1}{(1 - \varepsilon_1) / \varepsilon_1 A_1} = \frac{J_1 - J_2}{(A_1 F_{12})^{-1}} + \frac{J_1 - E_{b3}}{(A_1 F_{13})^{-1}}
\]

\[
q_2 = \frac{E_{b2} - J_2}{(1 - \varepsilon_2) / \varepsilon_2 A_2} = \frac{J_2 - J_1}{(A_2 F_{12})^{-1}} + \frac{J_2 - E_{b3}}{(A_2 F_{23})^{-1}}
\]

\[
E_{b1} = \sigma T_1^4 = (5.67 \times 10^{-8}) \times 1000^4 = 56700 \text{ W/m}^2
\]

\[
E_{b2} = \sigma T_2^4 = (5.67 \times 10^{-8}) \times 600^4 = 7348 \text{ W/m}^2
\]

\[
E_{b3} = J_3 = \sigma T_3^4 = (5.67 \times 10^{-8}) \times 300^4 = 459 \text{ W/m}^2
\]

\[
\begin{align*}
-10J_1 + 39J_2 &= -510582 \\
0.26J_1 - 1.67J_2 &= -7536
\end{align*}
\]

\[
J_1 = 99758 \text{ W/m}^2 \\
J_2 = 12487 \text{ W/m}^2
\]

Radiation Exchange Between Surfaces

\[
q_2 = \frac{E_{b2} - J_2}{(1 - \varepsilon_2) / \varepsilon_2 A_2} = \frac{7348 - 12487}{(1 - 0.5) / (0.5 \times 15)} = -77.1 \text{ kW}
\]