Heat Transfer

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Chapter 6

Introduction to Convection

Today’s Topics

- Boundary Layer Similarity
- Solution of the Boundary Layer Equations
- Physical Significance of Dimensionless Parameters
- The Reynolds Analogy
Boundary Layer Similarity

By defining the non-dimensional variables:

\[
x' = \frac{x}{L}, \quad y' = \frac{y}{L}, \quad \frac{u'}{V} = \frac{u}{V}, \quad \frac{v'}{V} = \frac{v}{V}, \quad T' = \frac{T - T_i}{T_e - T_i}
\]

\[
u \frac{\partial u'}{\partial x} + v \frac{\partial u'}{\partial y} = \frac{1}{\rho V^2} \frac{dP}{dx} + \frac{v}{VL} \frac{\partial^2 u'}{\partial y^2}
\]

\[
n' = n(0), \quad n' = n(\infty) = \frac{u(x', \infty)}{V}
\]

\[
u \frac{\partial T'}{\partial x} + v \frac{\partial T'}{\partial y} = \frac{\alpha}{VL} \frac{\partial^2 T'}{\partial y^2}
\]

\[
n' = n(0), \quad T'(x', \infty) = 1
\]

Boundary Layer Similarity

Similarity parameters are important because they allow us to apply results obtained for a surface experiencing one set of conditions to geometrically similar surfaces experiencing entirely different conditions. Two important similarity parameters are:

\[
\text{Re}_L = \frac{VL}{\nu} \quad \text{Reynolds Number}
\]

\[
\text{Pr} = \frac{\nu}{\alpha} \quad \text{Prandtl Number}
\]
Boundary Layer Similarity

From the non-dimensional momentum equation, we have:

\[ u^* = f \left( \frac{x^*, y^*, \text{Re}, \frac{dP}{dx}}{} \right) \]

\[ \tau_* = \mu \frac{\partial u}{\partial y} \bigg|_{y^*=0} = \left( \frac{\mu \nu}{L} \frac{\partial u}{\partial y} \right) \bigg|_{y^*=0} \]

\[ C_f = \frac{\tau_*}{\rho \nu^2 / 2} = \frac{2}{\text{Re}_L} \frac{\partial u}{\partial y} \bigg|_{y^*=0} \]

\[ \frac{\partial u}{\partial y} \bigg|_{y^*=0} = f \left( \frac{x^*, \text{Re}_L, \frac{dP}{dx}}{} \right) \]

\[ C_f = \frac{2}{\text{Re}_L} f \left( x^*, \text{Re}_L \right) \]

Boundary Layer Similarity

From the non-dimensional energy equation, we have:

\[ T^* = f \left( x^*, y^*, \text{Re}_L, \text{Pr}, \frac{dP}{dx} \right) \]

\[ h = -\frac{k_j}{L} \left( T_x - T_y \right) \frac{\partial T^*}{\partial y} \bigg|_{y^*=0} = \frac{k_j}{L} \frac{\partial T^*}{\partial y} \bigg|_{y^*=0} \]

\[ Nu = \frac{hL}{k_j} = \frac{\partial T^*}{\partial y} \bigg|_{y^*=0} \]

\[ Nu = \frac{hL}{k_j} = f \left( x^*, \text{Re}_L, \text{Pr} \right) \]

\[ \frac{Nu}{k_j} = f \left( \text{Re}_L, \text{Pr} \right) \]
Physical Significance of Dimensionless Parameters

\[ \text{Re}_L = \frac{\rho v L}{\mu} = \frac{\rho v^2}{L} \cdot \frac{F_{bem}}{F_{bemax}} \]

\[ \text{Pr} = \frac{v}{\alpha} \]

\[ \frac{\delta}{\delta_i} \approx \text{Pr}^n \]

- \( \text{Pr} \gg 1 \Rightarrow \delta \gg \delta_i \) liquids
- \( \text{Pr} \approx 1 \Rightarrow \delta = \delta_i \) gases
- \( \text{Pr} \ll 1 \Rightarrow \delta << \delta_i \) liquid metals

The Reynolds Analogy

For flow over a flat plate with \( \frac{dP}{dx} = 0 \) and gases flow, where \( \text{Pr} \approx 1 \)

\[ u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{1}{\text{Re}_L} \frac{\partial^2 u}{\partial y^2} \]

\[ u^+(x',0) = v^+(x',0) = 0 \quad u^+(x',\infty) = 1 \]

\[ u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{1}{\text{Re}_L} \frac{\partial^2 T}{\partial y^2} \]

\[ T^+(x',0) = 0 \quad T^+(x',\infty) = 1 \]

Hence, the functional forms of the solutions for \( u^+ \) and \( T^+ \) must be equivalent.

\[ \frac{C_f}{2} = Nu \quad \text{St} = \frac{h}{\rho v C_p} = \frac{Nu}{\text{Re} \text{Pr}} \]

\[ \frac{C_f}{2} = \text{St} \frac{Pr_{10}}{Pr} \]

For the case and \( \text{Pr} \neq 1 \) we can correct the Reynolds analogy as:

\[ \frac{C_f}{2} = \text{St} \frac{Pr_{10}^{1/3}}{Pr} \]

(0.6 < Pr < 60) (Colburn J factor)