Heat Transfer

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Chapter 8

Convection in Internal Flows
Today’s Topics

- Convection in Non-Circular Ducts
- Heat Transfer Enhancement
- Thermal Analysis in Turbulent Flows
Convection in Non-Circular Ducts

All the previous correlations can be used by defining a Hydraulic Diameter

\[ D_h = \frac{4A_c}{P} \]

This approach is accurate for turbulent flows \( Re_D \geq 2300 \) and \( Pr \geq 0.7 \)

But this approach is less accurate for laminar flows.

For laminar fully developed flows \( \text{Table 8.1} \)
### Table 3.1  Nusselt numbers and friction factors for fully developed laminar flow in tubes of differing cross section

<table>
<thead>
<tr>
<th>Cross Section</th>
<th>$\frac{b}{a}$</th>
<th>$Nt_m = \frac{hD_n}{k}$</th>
<th>(Uniform $q_n^{\prime}$)</th>
<th>(Uniform $T_n$)</th>
<th>$f , Re_{D_n}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.0</td>
<td>4.36</td>
<td>3.66</td>
<td>2.98</td>
<td>57</td>
</tr>
<tr>
<td></td>
<td>1.43</td>
<td>3.73</td>
<td>3.08</td>
<td></td>
<td>59</td>
</tr>
<tr>
<td></td>
<td>2.0</td>
<td>4.12</td>
<td>3.39</td>
<td></td>
<td>62</td>
</tr>
<tr>
<td></td>
<td>3.11</td>
<td>4.79</td>
<td>3.96</td>
<td></td>
<td>69</td>
</tr>
<tr>
<td></td>
<td>4.0</td>
<td>5.33</td>
<td>4.44</td>
<td></td>
<td>73</td>
</tr>
<tr>
<td></td>
<td>8.0</td>
<td>6.49</td>
<td>5.60</td>
<td></td>
<td>82</td>
</tr>
<tr>
<td>Heated</td>
<td>$\infty$</td>
<td>8.23</td>
<td>7.54</td>
<td></td>
<td>96</td>
</tr>
<tr>
<td>Insulated</td>
<td>$\infty$</td>
<td>5.39</td>
<td>4.86</td>
<td></td>
<td>96</td>
</tr>
<tr>
<td></td>
<td>1.0</td>
<td>3.11</td>
<td>2.49</td>
<td></td>
<td>53</td>
</tr>
</tbody>
</table>

Convection in Non-Circular Ducts

Concentric tube annulus

\[ q_i^\prime = h_i (T_{s,i} - T_m) \]
\[ q_o^\prime = h_o (T_{s,o} - T_m) \]

\[ Nu_i \equiv \frac{h_i D_h}{k} \]
\[ Nu_o \equiv \frac{h_o D_h}{k} \]

\[ D_h = \frac{4(\pi/4)(D_o^2 - D_i^2)}{\pi D_o + \pi D_i} = D_o - D_i \]
Convection in Non-Circular Ducts

For the case of fully developed laminar flow with one surface insulated

### Table 8.2  
Nusselt number for fully developed laminar flow in a circular tube annulus with one surface insulated and the other at constant temperature

<table>
<thead>
<tr>
<th>$D_i/D_o$</th>
<th>$Nu_i$</th>
<th>$Nu_o$</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>—</td>
<td>3.66</td>
<td>See Equation 8.55</td>
</tr>
<tr>
<td>0.05</td>
<td>17.46</td>
<td>4.06</td>
<td></td>
</tr>
<tr>
<td>0.10</td>
<td>11.56</td>
<td>4.11</td>
<td></td>
</tr>
<tr>
<td>0.25</td>
<td>7.37</td>
<td>4.23</td>
<td></td>
</tr>
<tr>
<td>0.50</td>
<td>5.74</td>
<td>4.43</td>
<td></td>
</tr>
<tr>
<td>$\approx 1.00$</td>
<td>4.86</td>
<td>4.86</td>
<td>See Table 8.1, $b/a \rightarrow \infty$</td>
</tr>
</tbody>
</table>
Convection in Non-Circular Ducts

If uniform heat flux conditions exist at both surfaces,

\[ Nu_i = \frac{N_{U_{ii}}}{1 - (q_o''/q_i'')\theta_i^*} \quad \text{and} \quad Nu_o = \frac{N_{U_{oo}}}{1 - (q_i''/q_o'')\theta_o^*} \]

**Table 8.3** Influence coefficients for fully developed laminar flow in a circular tube annulus with uniform heat flux maintained at both surfaces

<table>
<thead>
<tr>
<th>( D_i/D_o )</th>
<th>( N_{U_{ii}} )</th>
<th>( N_{U_{oo}} )</th>
<th>( \theta_i^* )</th>
<th>( \theta_o^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>—</td>
<td>4.364</td>
<td>( \infty )</td>
<td>0</td>
</tr>
<tr>
<td>0.05</td>
<td>17.81</td>
<td>4.792</td>
<td>2.18</td>
<td>0.0294</td>
</tr>
<tr>
<td>0.10</td>
<td>11.91</td>
<td>4.834</td>
<td>1.383</td>
<td>0.0562</td>
</tr>
<tr>
<td>0.20</td>
<td>8.499</td>
<td>4.833</td>
<td>0.905</td>
<td>0.1041</td>
</tr>
<tr>
<td>0.40</td>
<td>6.583</td>
<td>4.979</td>
<td>0.603</td>
<td>0.1823</td>
</tr>
<tr>
<td>0.60</td>
<td>5.912</td>
<td>5.099</td>
<td>0.473</td>
<td>0.2455</td>
</tr>
<tr>
<td>0.80</td>
<td>5.58</td>
<td>5.24</td>
<td>0.401</td>
<td>0.299</td>
</tr>
<tr>
<td>1.00</td>
<td>5.385</td>
<td>5.385</td>
<td>0.346</td>
<td>0.346</td>
</tr>
</tbody>
</table>
Convection in Non-Circular Ducts

For fully-developed turbulent flow, the influence coefficients are a function of \( Re \) and \( Pr \). However, to a first approximation \( h_i \) and \( h_o \) may be assumed to equal.

\[
Nu_D = 0.023 \left( \frac{Re_D}{Pr} \right)^{4/5}, \quad \begin{cases} 
n = 0.4 & \text{for heating} \\
n = 0.3 & \text{for cooling} \end{cases}
\]
Figure 8.12  Internal flow heat transfer enhancement schemes: (a) longitudinal section and end view of coil-spring wire insert, (b) longitudinal section and cross-sectional view of twisted tape insert, (c) cut-away section and end view of longitudinal fins, and (d) longitudinal section and end view of helical ribs.
Heat Transfer Enhancement

Figure 8.13 Schematic of helically coiled tube and secondary flow in enlarged cross-sectional view.
Heat Transfer Enhancement

\[ Re_{D,c,h} = Re_{D,c}[1 + 12(D/C)^{0.5}] \]

For fully developed laminar flow with \( C/D \geq 3 \), the friction factor is

\[ f = \frac{64}{Re_D} \quad Re_D(D/C)^{1/2} \leq 30 \]

\[ f = \frac{27}{Re_D^{0.725}}(D/C)^{0.1375} \quad 30 \leq Re_D(D/C)^{1/2} \leq 300 \]

\[ f = \frac{7.2}{Re_D^{0.5}}(D/C)^{0.25} \quad 300 \leq Re_D(D/C)^{1/2} \]
Heat Transfer Enhancement

\[
N u_D = \left[ \left( 3.66 + \frac{4.343}{a} \right)^3 + 1.158 \left( \frac{Re_D(D/C)^{1/2}}{b} \right)^{3/2} \right]^{1/3} \left( \frac{\mu}{\mu_s} \right)^{0.14}
\]

\[
a = \left( 1 + \frac{927(C/D)}{Re_D^2 Pr} \right) \quad \text{and} \quad b = 1 + \frac{0.477}{Pr}
\]

\[
\left[ 0.005 \leq Pr \leq 1600 \right] \quad \left[ 1 \leq Re_D(D/C)^{1/2} \leq 1000 \right]
\]
Sample Problem

**Known:** Heat rate per unit length at the inner surface of an annular recuperator. Flow rate and inlet temperature of air passing through annular region.

**Find:** (a) Temperature of air leaving the recuperator, (b) Inner pipe temperature at inlet and outlet pipe temperature at inlet

\[ q_i' = 1.25 \times 10^5 \text{W/m} \]
\[ m_a = 2.1 \text{kg/s} \]
\[ D_i = 2 \text{m} \]
\[ D_o = 2.05 \text{m} \]
\[ c_{p,a} = 1030 \text{J/kg.K} \]
\[ Pr = 0.68 \]
\[ \mu_a = 270 \times 10^{-7} \text{N.s/m}^2 \]
\[ k_a = 0.041 \text{W/m.K} \]
\[ T_{a,1} = 300 \text{K} \]
\[ L = 7 \text{m} \]
Sample Problem

\[ q' L = \dot{m}_a c_{p,a} (T_{a,2} - T_{a,1}) \Rightarrow T_{a,2} = T_{a,1} + \frac{q' L}{\dot{m}_a c_{p,a}} \]

\[ \Rightarrow T_{a,2} = 300 + \frac{1.25 \times 10^5 \times 7}{2.1 \times 1030} = 704.5 \, K \]

\[ \text{Re}_D = \frac{\rho u_m D_h}{\mu} = \frac{\dot{m}_a (D_o - D_i)}{\pi / 4(D_o - D_i)} = \frac{4\dot{m}_a}{\pi(D_o - D_i)\mu} \]

\[ \text{Re}_D = \frac{4 \times 2.1}{\pi \times 4.05 \times 270 \times 10^{-7}} = 24452 \, (turbulent) \]

\[ Nu_i \approx Nu_o = 0.023 \, \text{Re}_D^{4/5} \, \text{Pr}^{0.4} = 0.023 \times 24452^{4/5} \times 0.68^{0.4} = 63.88 \]

\[ h_i \approx h_o = Nu \times k / h = 63.88 \times 0.041 / 0.05 = 52.38 \, W / m^2K \]
Sample Problem

\[ q = q_i' L = h_i A (T_{s,i} - T_m) \Rightarrow \]

\[ T_{s,i,1} = T_{a,1} + \frac{q_i' L}{h_i (\pi D_i L)} = 300 + \frac{1.25 \times 10^5 \times 7}{52 \times \pi \times 2 \times 7} = 682.5 \text{K} \]

\[ T_{s,i,2} = T_{a,2} + \frac{q_i' L}{h_i (\pi D_i L)} = 704.5 + \frac{1.25 \times 10^5 \times 7}{52 \times \pi \times 2 \times 7} = 1087 \text{K} \]