Problem 1)

Suppose we have a distillation process where the objective is to separate components of a mixture in the input stream. The process is pictured in Figure P5.14. The relationship between the input variable, temperature, and the output variable, distillate fractions, is not precise but the human operator of this process has developed an intuitive understanding of this relationship. The universe for each of these variables is

$$\begin{aligned} X &= \text{universe of temperatures (degree fahrenheit)} \\ &= \{160, 165, 170, 175, 180, 185, 190, 195\}. \end{aligned}$$

$$Y = \text{universe of distillate fractions (percentage)} = \{77, 80, 83, 86, 89, 92, 95, 98\} .$$

Now we define fuzzy sets $\underset{\sim}{A}$ and $\underset{\sim}{B}$ on X and Y, respectively:

$$\underset{\sim}{A} = \text{temperature of input steam is hot} = \left\{ \frac{0}{175} + \frac{0.7}{180} + \frac{1}{185} + \frac{0.4}{190} \right\}.$$

$$\underset{\sim}{B} = \text{separation of mixture is good} = \left\{ \frac{0}{89} + \frac{0.5}{92} + \frac{0.8}{95} + \frac{1}{98} \right\}.$$

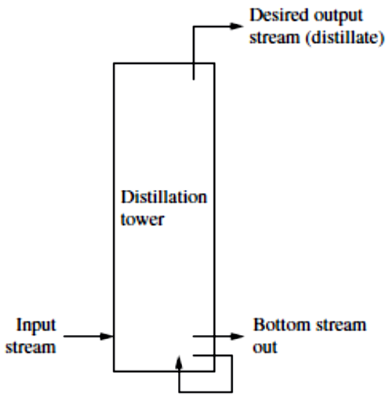
We wish to determine the proposition, IF "temperature is hot" THEN "separation of mixture is good," or symbolically, $\underset{\sim}{A} \rightarrow \underset{\sim}{B}$. From this,

(a) Find $\underset{\sim}{R} = (\underset{\sim}{A} \times \underset{\sim}{B}) \cup (\overline{\underset{\sim}{A}} \times Y)$.

(b) Now define another fuzzy linguistic variable as

$$\underset{\sim}{A}' = \left\{ \frac{1}{170} + \frac{0.8}{175} + \frac{0.5}{180} + \frac{0.2}{185} \right\},$$

and for the "new" rule IF $\underset{\sim}{A}'$ THEN $\underset{\sim}{B}'$ find $\underset{\sim}{B}'$ using max–min composition, that is, find $\underset{\sim}{B}' = \underset{\sim}{A}' \circ \underset{\sim}{R}$.



Problem 2)

The calculation of the vibration of an elastic structure depends on knowing the material properties of the structure as well as its support conditions. Suppose we have an elastic structure, such as a bar of known material, with properties like wave speed (C), modulus of elasticity (E), and cross-sectional area (A). However, the support stiffness is not well-known; hence the fundamental natural frequency of the system is not precise either. A relationship does exist between them, though, as illustrated in Figure P5.15.

RE P5.15

Define two fuzzy sets,

$$\underset{\sim}{K} = \text{“support stiffness” (pounds per square inch)},$$

$$\underset{\sim}{K}_1 = \text{“first natural frequency” of the system (hertz)},$$

with membership functions

$$\underset{\sim}{K} = \left\{ \frac{0}{1e+3} + \frac{0.2}{1e+4} + \frac{0.5}{1e+5} + \frac{0.8}{5e+5} + \frac{1}{1e+6} + \frac{0.8}{5e+6} + \frac{0.2}{1e+7} \right\}.$$

$$\underset{\sim}{f}_1 = \left\{ \frac{0}{100} + \frac{0}{200} + \frac{0.2}{500} + \frac{0.5}{800} + \frac{1}{1000} + \frac{0.8}{2000} + \frac{0.2}{5000} \right\}.$$

(a) Using the proposition, IF x is $\underset{\sim}{K}$, THEN y is $\underset{\sim}{f}_1$, find this relation using the following forms of the implication $\underset{\sim}{K} \rightarrow \underset{\sim}{f}_1$:

(i) Classical $\mu_R = \max[\min(\mu_K, \mu_{f_1}), (1 - \mu_K)]$
(ii) Mamdani $\mu_R = \min(\mu_K, \mu_{f_1})$
(iii) Product $\mu_R = \mu_K \cdot \mu_{f_1}$

(b) Now define another antecedent, say $\underset{\sim}{K}'$ = “damaged support,”

$$\underset{\sim}{K}' = \left\{ \frac{0}{1e+3} + \frac{0.8}{1e+4} + \frac{0.1}{1e+5} \right\}.$$

Find the system’s fundamental (first) natural frequency due to the change in the support conditions, that is, find $\underset{\sim}{f}_1$ = “first natural frequency due to damaged support” using classical implication from part (a), subpart (i) preceding, and

(i) max–min composition
(ii) max–product composition.

Problem 3)

For steel design, the cross-sectional area to column-height ratio largely determines the susceptibility of the columns to buckling under axial loads. The normalized ratios are on the universe, X = {0, 1, 2, 3}. These ratios are characterized as "small" to "large":

$$\text{"Small"} = \left\{ \frac{1}{0} + \frac{0.9}{1} + \frac{0.8}{2} + \frac{0.7}{3} \right\}.$$

$$\text{"Large"} = \left\{ \frac{0}{0} + \frac{0.1}{1} + \frac{0.2}{2} + \frac{0.3}{3} \right\}.$$

Calculate the membership functions for the following phrases:

(a) very small

(b) fairly small ($= [\text{small}]^{2/3}$)

(c) very, very large

(d) not fairly large and very, very small.

Problem 4)

Calculate the defuzzify value of *Z* using four defuzzification methods given in the book.