Problem 5-1

The code period is $2^{11} - 1 = 2047$. The serial search steps are 1/2 chip so a total of 4094 phase steps must be searched in one complete code cycle. The post despreading filter bandwidth is 10,000 Hz to accommodate frequency uncertainty. The signal-to-noise ratio at the post-despreading filter output is approximately 10 dB - 10 $\log_{10}(10{,}000/100)$ = -10 dB, where the data bandwidth is assumed to be 100 Hz. The signal-to-noise ratio is $P/B_N N_0$ where B_N = 10,000 Hz. Thus P/N_0 = 30 dB-Hz. The nomogram of Figure 5-11 could be used by trial and error to find $B_N T_i$ and the threshold to yield $P_d = 0.8$ and $P_{fa} = 10^{-3}$. It would be found that the $B_N T_i$'s required are too large for the nomogram. Thus, the Gaussian approximation for the integrator output pdf is valid. The mean of the output is given by (5-73) and its variance is found from (5-85). If V is the integrator output,

$$E(V) = P + 2\sigma_n^2$$

where σ_n^2 is the variance of either $n_I(t)$ or $n_Q(t)$; i.e.,

$$\sigma_n^2 = \frac{B_N N_0}{2}$$

Also, the variance of V is

$$\sigma_V^2 = \frac{2PN_0 B_N + (N_0 B_N)^2}{B_N T_i}$$

When an incorrect code phase is being evaluated,

$$E\{V\} = 2\sigma_n^2 = B_N N_0$$

and

$$\sigma_V^2 = \frac{N_0 B_N}{T_i}$$

Hence

$$P_{fa} = \int_{V_T}^{\infty} \frac{e^{-(\alpha - B_N N_0)^2/2\sigma_V^2}}{\sqrt{2\pi\sigma_V^2}} d\alpha = Q\left[\frac{\sqrt{B_N T_i}(V_T - B_N N_0)}{N_0 B_N}\right]$$

If the corrrect code phase is being evaluated,

$$P_d = Q\left[\frac{(V_T - P - B_N N_0)\sqrt{B_N T_i}}{\sqrt{2PN_0 B_N + (B_N N_0)^2}}\right]$$

Arbitrarily assume $P = 1$ which implies $N_0 = 10^{-3}$. The false alarm and detection probabilities are

$$P_{fa} = Q[(V_T - 10)(10\sqrt{T_i})] = 10^{-3} \text{ and } P_d = Q[(V_T - 11)(9.13\sqrt{T_i})] = 0.8$$

Using the rational approximation to the Q-function and trial and error, we find $V_T = 10.77$ and $T_i = 0.165$ (a computer mathematics package such as Mathcad is particularly useful here). The average acquisition time is found using (5-12):

$$\overline{T_s} = (C - 1)T_{da}\left(\frac{2 - P_d}{2P_d}\right) + \frac{T_i}{P_d}$$

with

$$T_{da} = T_i + T_{fa}P_{fa} = 0.165 + 16.5\times 10^{-3} = 0.182$$

Therefore

$$\overline{T_s} = (4094 - 1)(0.182)\left(\frac{2 - 0.8}{2(0.8)}\right) + \frac{0.165}{0.8} = 557 \text{ s}$$

One search cycle through the code requres (4094)(0.165) s. The probability of acquisition on this single pass is 0.8. The probability of having acquired after two passes through the code is

$$P_{d_2} = 0.8 + (1 - 0.8)(0.8) = 0.96$$

After three passes, the probability of having acquired is

$$P_{d_3} = 0.8 + (1 - 0.8)(0.8) + (1 - 0.8)^2(0.8) = 0.992$$

The correct phase is uniforn1ly distributed over all possible code passes so that the cumulative probability of acquisition is linearly increasing over each pass through the code, reaching the P_{dJ} just calculated at the end of each pass.

Problem 6-1

The bit error probability for barrage noise jamming of BPSK/BPSK modulation is given by (6-12):

$$P_b = Q\left[\sqrt{\frac{2}{K[N_0 R/P + JR/PW]}}\right]$$

where the factor K is defined in (6-8) and accounts for noise being bandlimited to about the spread signal bandwidth and therefore being reduced in magnitude due to despreading. Assume the barrage noise bandwidth equals the null-to-null spread signal bandwidth. Then $K = 0.903/2$. Assume negligible thermal noise. Then

$$P_b = Q\left[\sqrt{\frac{2}{0.451\,(J/P)[75/20\times 10^6]}}\right] = Q\left[\sqrt{1.18\times 10^6 \frac{P}{J}}\right].$$

Solve by trial and error for P/J to give $P_b = 10^{-2}$ (P/J = -53.5 dB) and $P_b = 10^{-5}$ (P/J = -48.12 dB). The bit error probability for worst case pulse noise jamming is given by (6-55):

$$(P_b)_{max} = \begin{cases} \dfrac{Q\left[1/\sqrt{K}\right]}{2(P/J)(W/R)}, & 0.5 < \dfrac{P}{J}\dfrac{W}{R} \\ Q\left[\sqrt{\dfrac{2}{K}\dfrac{P}{J}\dfrac{W}{R}}\right], & \dfrac{P}{J}\dfrac{W}{R} < 0.5 \end{cases}$$

where $K = 0.903$. Solve by trial and error to give $P_b = 10^{-2}$ (P/J = -45.6 dB) and $P_b = 10^{-5}$ (P/J = -15.6 dB). Observe that the pulsed jammer is 7.88 dB more effective than the barrage noise jammer at $P_b = 10^{-2}$ and 32.5 dB more effective at $P_b = 10^{-5}$.

Problem 6- 7

The image rejection filter must pass the entire spread signal. A bandwidth much wider than necessary will permit out-of-band interferers to pass. Therefore, make the image rejection filter bandwidth about 1024x200 kHz = 204.8 MHz. The post -despreading bandwidth must pass the data modulation. Therefore, use a bandwidth of l00 kHz + 2xO.5x 100 kHz = 200 kHz. The jammer power is 10 times as large as the signal power. The optimum jamming strategy places a single interferer in any hop band. The power of each interferer should equal the signal power. Thus, the optimal number of jamming tones is $J/P = 10P/P = 10$. The tone spacing is 200 kHz (1 per band). The jammer tone power level is $J_q = J/q = 10P/10 = P$. The frequency hop rate is arbitrary except for requiring that the system must be a slow hopper. It must be such because the hop spacing is determined by the data bandwidth and not by the duration of the hop chip. In order that hop transitions not affect performance, choose a hop rate equal to 1/10 of the data rate or 10 khops/s. The bit error probability is calculated by noting that an error is made with probability 112 on the hops which are jammed, and no errors are made on the remaining 1024- 10 = 1014 hop tones. Thus,

$$\overline{P}_b = \frac{1014}{1024} \times 0 + \frac{10}{1024} \times 0.5 = 4.9 \times 10^{-3}$$