Problem 4-4

The tracking phase jitter for the noncoherent delay lock tracking loop is given by

$$\sigma^2 = \frac{\eta W_L}{2K_d^2}$$

where $\eta/2$ is the equivalent noise power spectral density at the output of the phase detector. From (4-75)

$$\frac{\eta}{2} = \frac{1}{2}(K_1N_0)^2B_N + \frac{1}{2}K_1^2N_0P\left\{R_c^2\left[\left(\delta - \frac{\Delta}{2}\right)T_c\right] + R_c^2\left[\left(\delta - \frac{\Delta}{2}\right)T_c\right]\right\}$$

The autocorrelation functions are evaluated at $\delta = 0$ to give

$$R_c^2\left(\pm\frac{\Delta}{2}T_c\right) = \left(1 - \frac{\Delta}{2}\right)^2$$

Thus,

$$\frac{\eta}{2} = \frac{1}{2}(K_1N_0)^2B_N + K_1^2N_0P\left(1 - \frac{\Delta}{2}\right)^2$$

From Fig. 4-13 with $N >> 1$,

$$K_d = 2K_1P\left(1 - \frac{\Delta}{2}\right)$$

Hence

$$\sigma^2 = \frac{\eta}{2K_d^2}W_L = \left[\frac{N_0^2B_N}{8(1 - \Delta/2)^2P^2} + \frac{N_0P}{4P^2}\right]W_L$$

$$= \frac{N_0W_L}{4P}\left[1 + \frac{N_0B_N}{2P(1 - \Delta/2)^2}\right] = \frac{1}{2\rho_L}\left[1 + \frac{1}{2(1 - \Delta/2)^2\rho_{IF}}\right]$$

Since the IF bandwidth is 100 times larger than the two-sided loop bandwidth, $\rho_{IF} = \rho_L/200$ and

$$\sigma^2 = \frac{1}{2\rho_L}\left[1 + \frac{100}{2(1 - \Delta/2)^2\rho_L}\right]$$

Problem 4-8

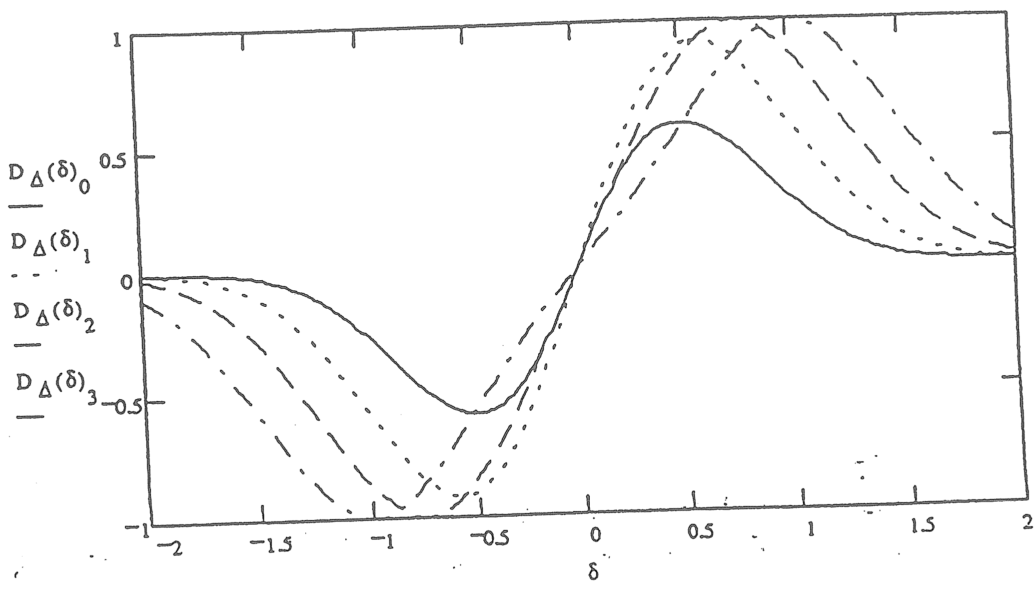
For MSK spreading modulation, the autocorrelation function of the complex envelope is [see (4-141)]

$$R_\theta(\tau) = \frac{1}{2\pi}\left[\pi\left(1 - \frac{|\tau|}{2T_c}\right)\cos\left(\frac{\pi|\tau|}{2T_c}\right) + \sin\left(\frac{\pi|\tau|}{2T_c}\right)\right]$$

The discriminator function using this autocorrelation function is

$$D_\Delta(\delta) = 4R_\theta^2\left[\left(\delta - \frac{\Delta}{2}\right)T_c\right] - 4R_\theta^2\left[\left(\delta + \frac{\Delta}{2}\right)T_c\right]$$

This is plotted below for $\Delta = 0.5$ (solid line), 1.0 (short dashes), 1.5 (long dashes), and 2.0 (alternating short and long dashes).



Problem 4-11

Equation (4-26) gives the expression for the normalized tracking error:

$$\frac{T_d(s) - \hat{T}_d(s)}{T_c} = \frac{T_d(s)}{T_c}\,\frac{s^2 + s/\tau_1}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

where

$$\omega_n = \sqrt{K_c g_c/\tau_1} \quad \text{and} \quad \zeta = \frac{\tau_2}{2\omega_n} + \frac{1}{2\tau_1\omega_n}$$

(a) For this case, $T_d(t)/T_c = A\,u(t)$ giving $T_d(s)/T_c = A/s$. Thus

$$E(s) = \frac{A(s + 1/\tau_1)}{s^2 + 2\zeta\omega_n s + \omega_n^2} = A\frac{(s + \zeta\omega_n) + (1/\tau_1 - \zeta\omega_n)}{(s + \zeta\omega_n)^2 + \omega_n^2(1 - \zeta^2)}$$

The inverse Laplace transform is

$$e(t) = Ae^{-\zeta\omega_n t}\left\{\cos\left[\sqrt{\omega_n^2(1 - \zeta^2)}\,t\right] + \frac{1/\tau_1 - \zeta\omega_n}{\sqrt{\omega_n^2(1 - \zeta^2)}}\sin\left[\sqrt{\omega_n^2(1 - \zeta^2)}\,t\right]\right\},\ T \geq 0$$

(b) For a ramp of slope B, $T_d(s)/T_c = B/s^2$ which gives for $E(s)$ the following:

$$E(s) = \frac{B(s + 1/\tau_1)}{s(s^2 + 2\zeta\omega_n s + \omega_n^2)} = B\frac{\omega_n\sqrt{1 - \zeta^2}/\omega_n\sqrt{1 - \zeta^2}}{(s + \zeta\omega_n)^2 + \omega_n^2(1 - \zeta^2)} + \frac{B/\tau_2}{s(s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$= \frac{B}{\omega_n\sqrt{1 - \zeta^2}}\,\frac{\omega_n\sqrt{1 - \zeta^2}}{(s + \zeta\omega_n)^2 + \omega_n^2(1 - \zeta^2)} + \frac{B}{\tau_2\omega_n^2}\left[\frac{1}{s} - \frac{s + 2\zeta\omega_n}{(s + \zeta\omega_n)^2 + \omega_n^2(1 - \zeta^2)}\right]$$

The inverse Laplace transform is

$$e(t) = \frac{B}{\omega_n\sqrt{1 - \zeta^2}}e^{-\zeta\omega_n t}\sin(\sqrt{1 - \zeta^2}\,\omega_n t)$$

$$+ \frac{B}{\tau_2\omega_n^2}\{1 - e^{-\zeta\omega_n t}[\cos(\sqrt{1 - \zeta^2}\,\omega_n t) + \frac{\zeta}{\sqrt{1 - \zeta^2}}\sin(\sqrt{1 - \zeta^2}\,\omega_n t)]\},\ t \geq 0$$

(c) The input $T_d(t)/T_c = Ct^2u(t)$ has Laplace transform $T_d(s)/T_c = 2C/s^3$. Partial fraction expansion gives the result

$$e(t) = \frac{2C}{\omega_n^2}\{1 - e^{-\zeta\omega_n t}[\cos(\sqrt{1 - \zeta^2}\,\omega_n t) + \frac{\zeta}{\sqrt{1 - \zeta^2}}\sin(\sqrt{1 - \zeta^2}\,\omega_n t)]$$

$$+ \frac{2C}{\tau_1\omega_n^2}\left[t + \frac{2\zeta}{\omega_n} + \frac{2\zeta}{\omega_n}e^{-\zeta\omega_n t}\cos(\sqrt{1 - \zeta^2}\,\omega_n t) + \frac{2\zeta^2 - 1}{\sqrt{1 - \zeta^2}}\sin(\sqrt{1 - \zeta^2}\,\omega_n t)\right],\ t \geq 0$$